



Dynamics and global geometry of manifolds without conjugate points

Rafael O. Ruggiero

Abstract. Manifolds with no conjugate points are natural generalizations of manifolds with nonpositive sectional curvatures. They have in common the fact that geodesics are global minimizers, a variational property of geodesics that is quite special. The restriction on the sign of the sectional curvatures of the manifold leads to a deep knowledge about the topology and the global geometry of the manifold, like the characterization of higher rank, nonpositively curved spaces as symmetric spaces. However, if we drop the assumptions concerning the local geometry of the manifold the study of geodesics becomes much harder. The purpose of this survey is to give an overview of the classical theory of manifolds without conjugate points where no assumptions are made on the sign of the sectional curvatures, since the famous work of Morse about minimizing geodesics of surfaces and the works of Hopf about tori without conjugate points. We shall show important classical and recent applications of many tools of Riemannian geometry, topological dynamics, geometric group theory and topology to study the geodesic flow of manifolds without conjugate points and its connections with the global geometry of the manifold. Such applications roughly show that manifolds without conjugate points are in many respects close to manifolds with nonpositive curvature from the topological point of view.