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Simplicité des groupes de transformations de surfaces

Abed Bounemoura

Abstract. The study of algebraic properties of groups of transformations of a manifold gives rise to an interplay between different areas of mathematics such as topology, geometry, dynamical systems and foliation theory. This volume is devoted to the question of simplicity of such groups, and we will mainly restrict our attention to the case where the manifold is a surface. In the first chapter, we will show that the identity component of the group of homeomorphisms of a closed surface is simple. This will lead us to the case of diffeomorphisms, and in the second chapter, we will give the complete proof of the Epstein-Herman-Mather-Thurston theorem stating that the group of C^∞-diffeomorphisms isotopic to the identity is also simple. We will also review the link with classifying spaces for foliations, and a result of Mather showing that the theorem remains true for C^k-diffeomorphisms, provided that k is different from $n+1$, where n is the dimension of the manifold. The last two chapters deal with conservative homeomorphisms and diffeomorphisms, by which we mean preserving a measure or a smooth volume or symplectic form. In those cases, there is a generalized rotation number showing that the associated groups cannot be simple. For conservative diffeomorphisms, the situation is well understood thanks to the work of Banyaga, but this is definitely not the case for conservative homeomorphisms of surfaces, and we will present some open problems in this direction as well as different attempts to solve them.

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