



# The remarkable effectiveness of ergodic theory in number theory

## Part I. Green-Tao theorem

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## Part II. Elkies-McMullen theorem

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**Abstract.** The main goal of this survey is the description of the fruitful interaction between Ergodic Theory and Number Theory via the study of two beautiful results: the first one by Ben Green and Terence Tao (about long arithmetic progressions of primes) and the second one by Noam Elkies and Curtis McMullen (about the distribution of the sequence  $\{\sqrt{n}\} \bmod 1$ ). More precisely, during the first part, we will see how the ergodic-theoretical ideas of Furstenberg about the famous Szemerédi theorem were greatly generalized by Green and Tao in order to solve the classical problem of finding arbitrarily long arithmetical progression of prime numbers, while the second part will focus on how Elkies and McMullen used the ideas of Ratner's theory (about the classification of ergodic measures related to unipotent dynamics) to compute *explicitly* the distribution of the sequence  $\{\sqrt{n}\}$  on the unit circle.

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