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Notes on the Dirichlet problem of a class of second order elliptic partial differential equations on a Riemannian manifold

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Abstract. In these notes we study the Dirichlet problem for critical points of a convex functional of the form

$$F(u) = \int_{\Omega} \phi\left(|\nabla u|\right),$$

where Ω is a bounded domain of a complete Riemannian manifold $\mathcal{M}$. We also study the asymptotic Dirichlet problem when $\Omega = \mathcal{M}$ is a Cartan-Hadamard manifold. Our aim is to present a unified approach to this problem which comprises the classical examples of the $p-$Laplacian ($\phi(s) = s^p$, $p > 1$) and the minimal surface equation ($\phi(s) = \sqrt{1+s^2}$). Our approach does not use the direct method of the Calculus of Variations which seems to be common in the case of the $p-$Laplacian. Instead, we use the classical method of a-priori C^1 estimates of smooth solutions of the Euler-Lagrange equation. These estimates are obtained by a coordinate free calculus. Degenerate elliptic equations like the $p-$Laplacian are dealt with by an approximation argument.

These notes address mainly researchers and graduate students interested in elliptic partial differential equations on Riemannian manifolds and may serve as a material for corresponding courses and seminars.

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