



# Real and complex Li-Sinai solutions of the 3D incompressible Navier-Stokes equations

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**Abstract.** In the framework of the Global Regularity Problem for the incompressible Navier-Stokes (NS) equations in the whole space  $\mathbb{R}^3$ , Li and Sinai in [*J. Eur. Math. Soc.*, 10:267–313, 2008] proved the existence of smooth complex solutions that become singular (“blow-up”) in a finite time. We report new results obtained by computer simulations on the behavior of complex solutions with support of Li-Sinai type and of real flows related to them. For the complex solutions the simulations indicate that the class of initial data leading to a blow-up is much larger than that considered by Li and Sinai. The real flows show some remarkable properties, such as a sharp increase of the total enstrophy and a concentration of high values of velocities and vorticity in small regions. We conclude with a discussion on the perspectives of a real blow-up in the framework of the Li-Sinai approach.

**Keywords.** Incompressible Navier-Stokes, singular solutions, blow-up, tornadoes.