# Preface

This volume is a collection of six closely related papers on the topology of knots and braids and their relations to quadratic forms, consisting of four new papers and two reprints.

Knot theory goes back to the 18th and the 19th centuries, pioneered by Alexandre-Théophile Vandermonde in France and Peter Guthrie Tait in Scotland. But the modern classification using algebra only took off in the 20th century. It was observed in the 1930s that any knotted curve in 3-space is the boundary of an oriented embedded surface, called a Seifert surface. The first homology of this surface is canonically equipped with an integral bilinear form $b$. Consider two closed oriented curves $x, y$ in the surface, push $x$ away from the surface in the positive normal direction to get a curve $x^+$, and define $b(x, y)$ to be the linking number of $x^+$ and $y$. One can try to study the topology of a knot through the algebraic invariants of the Seifert bilinear form. Unfortunately, this form is not symmetric and the Seifert surfaces are not unique, so one has to overcome some difficulties to obtain invariants for classification purposes. These ideas lead to numerical invariants called "signatures" of a knot, which originated in the algebraic theory of quadratic forms. This volume is a contribution to this aspect of the algebraic topology of knots, links and braids.

The first paper, by Étienne Ghys and Andrew Ranicki, is an extended survey of signatures in algebra, topology and dynamics, intended to serve as a general introduction to the subject. Some care has been taken in the first chapter to describe the historical development of the theory of quadratic forms.

The second paper, by Jean-Marc Gambaudo and Étienne Ghys on braids and signatures, was initially published ten years ago in the Bulletin de la Société Mathématique de France. We thought that it might be useful to include it in this volume since it can serve as a motivation for the other papers.

Arjeh Cohen and Jack van Wijk's contribution is also a reprint, from IEEE Transactions on Visualization and Computer Graphics, on how to visualize braids and their Seifert surfaces. Visualizing knots and links has always been of great help in topology and this paper contains pictures and algorithms for Seifert surfaces.

The last three papers discuss the Seifert bilinear form for knots and links and braids.

Julia Collins presents an algorithm for computing the Seifert matrix of a link from a braid representation.

Maxime Bourrigan deals with the relationship of the signatures of braids and quasimorphisms, developing further the work of Jean-Marc Gambaudo and Étienne Ghys.

Finally, Chris Palmer uses the algebraic theory of surgery to construct the Seifert matrix of a braid on the chain level.

Étienne Ghys and Andrew Ranicki
Guest editors