Entropy, Lyapunov exponents, and rigidity of group actions

Aaron Brown

with appendices by
Dominique Malicet, Davi Obata, Bruno Santiago and Michele Triestino,
Sébastien Alvarez and Mario Roldán

Edited by Michele Triestino

Abstract. This text is an expanded series of lecture notes based on a 5-hour course given at the workshop entitled *Workshop for young researchers: Groups acting on manifolds* held in Teresópolis, Brazil in June 2016. The course introduced a number of classical tools in smooth ergodic theory—particularly Lyapunov exponents and metric entropy—as tools to study rigidity properties of group actions on manifolds.

We do not present a comprehensive treatment of group actions or general rigidity programs. Rather, we focus on two rigidity results in higher-rank dynamics: the measure rigidity theorem for affine Anosov abelian actions on tori due to A. Katok and R. Spatzier and recent the work of the author with D. Fisher, S. Hurtado, F. Rodriguez Hertz, and Z. Wang on actions of lattices in higher-rank semisimple Lie groups on manifolds We give complete proofs of these results and present sufficient background in smooth ergodic theory needed for the proofs. A unifying theme in this text is the use of metric entropy and its relation to the geometry of conditional measures along foliations as a mechanism to verify invariance of measures.