Introduction to (generalized) Gibbs measures

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Abstract. In this monograph, we survey some key issues of the theory of Gibbsian and non-Gibbsian measures in finite-spin lattice systems. While non-Gibbsian measures are truly only the object of the last chapter, the material of the first chapters is selected with generalized Gibbs measures in mind. The topics of Gibbsian theory are then chosen for their foundational or contrasting role with respect to the measures analyzed in the final chapter, including in particular more detailed parts as e.g. the proof of the Choquet decomposition of Gibbs measures in Chapter 2, a proof of the Kozlov theorem in Chapter 3, under a slightly novel presentation that serves to introduce a telescoping procedure needed for generalized Gibbs measures in Chapter 5, and a careful discussion of the variational principle in Chapter 4. This monograph covers also the contents of mini-courses given in 2007 at the universities UFMG (Belo Horizonte) and UFRGS (Porto Alegre), whose aim was to convey, in a relatively short number of lectures, the heart of the theory needed to understand Gibbsianness and non-Gibbsianness.