



From particle systems to the BGK equation

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Abstract. In [Phys. Rev. **94** (1954), 511–525] the authors introduced a kinetic equation (the BGK equation), effective in physical situations where the Knudsen number is small compared to the scales where Boltzmann's equation can be applied, but not enough for using hydrodynamic equations. In this paper, we consider the stochastic particle system (inhomogeneous Kac model) underlying Bird's direct simulation Monte Carlo method (DSMC), with tuning of the scaled variables yielding kinetic and/or hydrodynamic descriptions. Although the BGK equation cannot be obtained from pure scaling, it does follow from a simple modification of the dynamics. This is proposed as a mathematical interpretation of some arguments in [Phys. Rev. **94** (1954), 511–525], complementing previous results in [Arch. Ration. Mech. Anal. **240** (2021), 785–808] and [Kinet. Relat. Models **16** (2023), 269–293].

Keywords. BGK equation, kinetic limits, stochastic particle systems.