SOCIEDADE BRASILEIRA DE MATEMÁTICA

ENSAIOS MATEMÁTICOS
2013, Volume **25**, 1–371

Parafermionic observables and their applications to planar statistical physics models

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Abstract. This volume is based on the PhD thesis of the author. Through the examples of the self-avoiding walk, the random-cluster model, the Ising model and others, the book explores in details two important techniques:

1. *Discrete holomorphicity and parafermionic observables*, which have been used in the past few years to study planar models of statistical physics (in particular their conformal invariance), such as *random-cluster models* and *loop $O(n)$-models.*

2. The *Russo-Seymour-Welsh theory* for percolation-type models with dependence. This technique was initially available for Bernoulli percolation only. Recently, it has been extended to models with dependence, thus opening the way to a deeper study of their critical regime.

The book is organized as follows. The first part provides a general introduction to planar statistical physics, as well as a first example of the parafermionic observable and its application to the computation of the connective constant for the self-avoiding walk on the hexagonal lattice.

2010 Mathematics Subject Classification: 60K35, 60F05, 60K37.

The second part deals with the family of random-cluster models. It studies the Russo-Seymour-Welsh theory of crossing probabilities for these models. As an application, the critical point of the random-cluster model is computed on the square lattice. Then, the parafermionic observable is introduced and two of its applications are described in detail. This part contains a chapter describing basic properties of the random-cluster model.

The third part is devoted to the Ising model and its random-cluster representation, the FK-Ising model. After a first chapter gathering the basic properties of the Ising model, the theory of s-holomorphic functions as well as Smirnov and Chelkak-Smirnov's proofs of conformal invariance (for these two models) are presented. Conformal invariance paves the way to a better understanding of the critical phase and the two next chapters are devoted to the study of the geometry of the critical phase, as well as the relation between the critical and near-critical phases.

The last part presents possible directions of future research by describing other models and several open questions.