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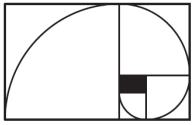

# Malliavin calculus and normal approximations

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**Abstract.** The goal of these notes is to introduce the basic elements of Malliavin calculus and to discuss its applications to quantitative normal approximations. The basic differential operators, the derivative and the divergence, are first introduced in the framework of the finite-dimensional Euclidean space equipped with the standard normal distribution. Later, we study these operators on the Wiener space and their relation with Wiener chaos expansions and the Ornstein-Uhlenbeck semigroup. Chapter 2 is devoted to the application of Malliavin calculus, combined with Stein's method, to derive estimations of the total variation distance in normal approximations. These estimates are applied, in Chapters 3 and 4, to the Breuer-Major theorem for stationary sequences and to spatial averaging of stochastic partial differential equations, respectively.

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**Keywords:** Malliavin calculus, Wiener chaos expansions, Ornstein-Uhlenbeck semigroup, Stein's method for normal approximations, total variation distance, central limit theorem, Breuer-Major theorem, fractional Brownian motion, fourth moment theorem, stochastic heat equation.