



# Navier-Stokes equation and Equivalence conjectures

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**Abstract.** General considerations on the Equivalence conjectures and a review of few mathematical results. The conjectures deal with properties of the NS equations regularized at ultraviolet (UV) cut off  $|\mathbf{k}| < N$  in the limit  $N \rightarrow \infty$ . (1) The UV-regularized NS equations with periodic b.c. are compared with similar reversible equations designed to obey, by modifying the usual friction term  $\nu\Delta\mathbf{u}$  into  $\alpha(\mathbf{u})\Delta\mathbf{u}$ , an extra conservation law (here the enstrophy,  $\mathcal{D}(\mathbf{u})$ , conservation). (2) The two equations are conjectured to assign, in the limit  $N \rightarrow \infty$  equal average expectation values to the class of observables  $O(\mathbf{u})$  which are local in the sense that they depend on finitely many (UV-cut-off  $N$ -independent) Fourier components of the velocity field  $\mathbf{u}$ . (3) A relevant *non local* observable is  $\alpha(\mathbf{u})$  and whether in its evolution  $\alpha(\mathbf{u}(t))$  becomes eventually  $> 0$  has been shown to imply uniform UV-independent bounds on all derivatives of  $\mathbf{u}$ , leading to think that positivity of  $\alpha$  only allows for large- $N$  deviations to negative values as  $N \rightarrow \infty$ . (4) For the non-local observables, defined by the spectrum of the Jacobian of the evolutions, approximate equivalence has been reported in a sense discussed here as well as a surprising, approximate, “pairing rule” for the Lyapunov exponents.

**Keywords.** Navier-Stokes equations, turbulence, Kolmogorov’s scaling, Ensembles equivalence, SRB distributions, reversibility.